Measuring the e/m ratio

In 1897, the physicist J.J. Thomson set out to prove that the cathode rays originating from a heated cathode were actually a stream of small negatively charged particles, now known as electrons. By studying the centripetal acceleration of electrons in a magnetic field, Thomson was able to successfully determine their charge-to-mass ratio. Thomson's work provided the first empirical evidence that atoms are not themselves fundamental building blocks of matter, paving the way for the field of subatomic physics. Thomson's discovery of the electron earnt him the Nobel Prize for Physics in 1906.

Safety

Electrical safety: All the signal voltages are small and harmless. The voltages in the mains-powered equipment is dangerous but are screened in normal use.

High Voltage: The beam tube requires dangerous contact voltages of up to 300 V. Please observe the following safety precautions:

- Only use the supplied connecting leads.
- Switch off all power supplies before altering the setup.
- Do not power up the experiment until the apparatus is fully assembled and cabled up.
- Do not handle the apparatus, particularly the Helmholtz coils, during operation.

Fragile equipment: The fine beam tube is an evacuated glass vessel and presents a danger of implosion.

- Do not subject the fine beam tube to mechanical stresses.
- Do not remove the tube from its holder.
- Take care with the plug at the glass base.

Background

The mass m of the electron is hard to measure experimentally. It is easier to determine the charge to mass ratio e/m, from which the mass m can be calculated if the elementary charge e is known. The e/m ratio can be determined experimentally by measuring the trajectory of electrons in an applied magnetic field.

1. How will electrons change their motion in the presence of a magnetic field? What path will the electrons follow in the field? Figure 1 below is provided to assist this question.

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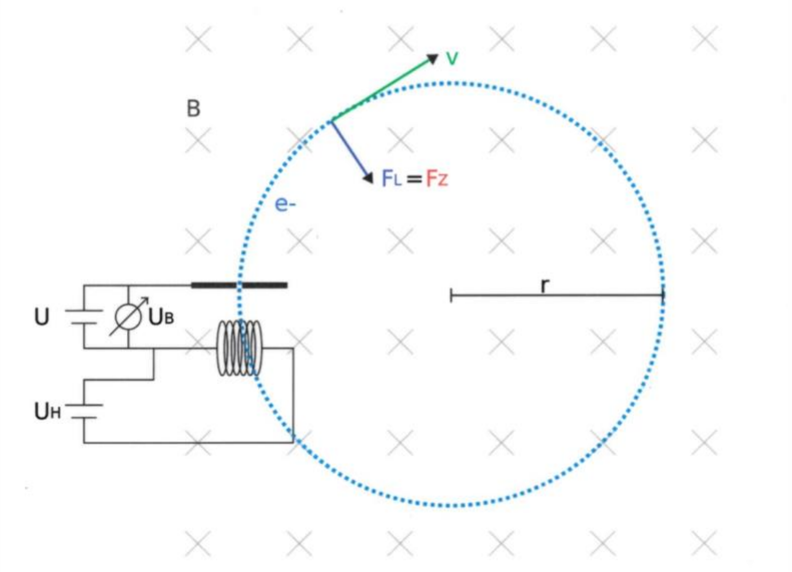

Figure 1: The trajectory of an electron in an applied magnetic field.

An electron moving at velocity v perpendicularly to a homogeneous magnetic field B, is subject to the *Lorentz force:*

$$F_{Lorenz} = evB$$

To make an electron move on a circular path, it needs a continuous *centripetal* force pulling it toward the center of the circle:

$$F_{Centripetal} = m\frac{v^2}{r}$$

The Lorentz force provides this continuous pull on the electron.

2. By equating the centripetal force to the Lorentz force, derive an expression for the charge-to-mass ratio e/m.

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Answer: $\frac{e}{m} = \frac{v}{Br}$

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In this experiment, the electrons are initially accelerated in a fine beam tube by applying a potential V. The resulting kinetic energy is

$$e\,V = \frac{1}{2} m\,v^2.$$

By substituting the velocity v into the above expression for the charge to mass ratio of the electron, we obtain:

$$\frac{e}{m} = \frac{2V}{(Br)^2}$$

The magnetic field B is generated by a pair of Helmholtz coils and is proportional to the current I in the coils as follows:

$$B = kI$$

The proportionality factor k has already been determined for our apparatus and is indicated on the apparatus.

Therefore the expression for the e/m ratio can be expressed as:

$$\frac{e}{m} = \frac{2V}{(Ikr)^2}$$

The fine beam tube contains hydrogen molecules at low pressure, which emit light through collisions with electrons. This makes the orbital motion of the electrons indirectly visible, and the radius r of this motion can be measured with a ruler. This in turn enables the e/m ratio to be calculated.

3. What uncertainties do you think affect the measurement of the orbital radius of the electron? What could you do to reduce these uncertainties?

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Task 1

- The apparatus should already be connected. If not, ask a member of staff to connect the leads.
- Turn all the dials to zero and switch on the power.
- Set the beam tube voltage to $V = 250$ [V] (**do not go higher than this**). Adjust the 50 [V] power supply to give a narrow, well-defined electron beam with clear edge definition.
- Adjust the DC power supply of the Helmholtz coils and look for the current I at which the electron beam is deflected into a closed orbit.

Question 1

Use the sliders and mirrors to measure the orbital radius of the electron beam. Care must be taken to reduce parallax uncertainty. Move the left slide so that its inner edge, mirror image, and the escape aperture of the electron beam all lay on the same line of sight. Now move the right slide so that its inner edge, mirror image, and the right-hand side of the circular electron beam orbit all lay on the same line of sight. Note that you can measure the diameter of the orbit to determine the radius.

Compare the circuit diagram on the apparatus with what is connected, and determine how you can vary and measure the beam potential V, and the Helmholtz coil current I. Experiment with different values of the potential V and current I and study how the radius r changes with these parameters.

1. What effects do you observe? How can you adjust these parameters to minimise the uncertainty in the radius r?

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Write down the value of k for this apparatus:

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Task 2

Measure the radius r for a number of different values of V and I.
From these measurements, use the formula for V to calculate e/m.
Remember that there will be an uncertainty on V, I, k and r.
Which has the highest uncertainty? Estimate the uncertainty in e/m.

V [V]	I [mA]	r [m]	e/m [C/kg]

The accepted value for the charge-to-mass ratio of the electron is
$\mathbf{e/m} = \mathbf{1.76} \times \mathbf{10^{11}}$ C/kg

1. How do your results compare to the accepted value? What is the percentage difference between your measured values of e/m and its accepted value?

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Question 2

A graphical method can be employed to measure the e/m ratio with an improved precision. The dependence of the accelerating potential V and the current I can be found by rearranging the equation for e/m and substituting for B:

$$V = 0.5\, k^2\, r^2 (e/m) \times I^2.$$

1. How can you graphically infer e/m using the formula above? (Hint: remember that k and r are fixed values in this task and that the theory predicts that V is proportional to I^2)

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Task 3

Position the left slide to align with the escape aperture, as before. Now set the right slide so the inside edges are separated by 80 mm, align the inside edge of the right slide with its mirror image, and adjust the coil current I until the electron beam runs tangentially along the slide edge covering the mirror image.

Reduce the potential V in steps of 250 [V] down to 150 [V], and adjust the coil current I so that the orbit of the electron beam has a diameter of 80 [mm]. Record your findings in the table below.

V [V]	I [A]	I^2 [A^2]
250		
225		
200		
175		
150		

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To assess your plot the $V(I^2)$ function for the chosen range of values. You may use the grid below or a computer program such as Excel.

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Question 4

1. Does your graph of V vs I^2 show a linear trend?

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2. Calculate the e/m ratio from the slope of the graph, including an estimate of the uncertainty. Compare your result with the accepted value.

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3. Was the experiment successful? How do you think you could improve your results?

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